Available at
www.ElsevierMathematics.com
POWERED BY SCIENCE DIRECT
Computational Statistics & Data Analysis 44 (2004) 669–691
COMPUTATIONAL STATISTICS & DATA ANALYSIS
www.elsevier.com/locate/csda

Bayesian sampling plans for exponential distribution based on uniform random censored data

Wen-Tao Huang[a,*], Yu-Pin Lin[b]

[a] Department of Management Sciences and Decision Making, Tamkang University, No. 151, Ying-Chuan Road, Tamsui 251, Taiwan, ROC

[b] Department of International Trade, Kuang Wu Institute of Technology, Taipei 112, Taiwan, ROC

Received 31 March 2001; accepted 31 October 2002

Abstract

The problem of a single sampling plan with polynomial loss for the exponential distribution based on uniformly distributed random censored data has been considered. A Bayes sampling plan is derived under various schemes of censoring time. It is specially focused on a quadratic loss and an unit time cost is included in the loss. Some optimal Bayes solutions are tabulated and some numerical comparisons between the proposed plan and a known plan under special loss are also made. It is shown that the optimal solutions of the known plan are not Bayes in general.

Keywords: Bayes sampling plan; Exponential population; Uniform random censoring

1. Introduction

Optimal sampling plan is one of the main research topics in quality control. Basically, there are two kinds of sampling plans, sampling for inspection by attributes and by variables. Many schemes such as the producer's and consumer's risk point schemes, defence sampling schemes, Dodge and Romig's schemes, and decision theoretic schemes have been proposed and studied, and they are used to choose a single sampling plan (see e.g. Wetherill, 1977). From the economical point of view, the decision theoretic schemes are considered to be more scientific and are therefore widely

* Corresponding author. Tel.: +866-2-26215656x2186; fax: +886-2-86313214.
E-mail address: 005697@mail.tku.edu.tw (W.-T. Huang).

0167-9473/03/$ - see front matter
PII: S0167-9473(02)00330-4

Table 1
Under FCT, optimal solutions (n_{B_1}, δ_{B_1}) and its Bayes risks

α	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$	β	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
1.5	4	1.9407	32.1416	1.0	0	∞	50.0000
2.0	2	0.9368	37.3428	1.25	1	1.6868	48.4154
2.5	2	1.2717	42.0310	1.5	3	2.7749	48.3673
3.0	2	1.6063	44.8481	2.0	2	1.6063	44.8481
3.5	2	1.9407	46.7695	2.5	2	1.1063	43.1733
4.0	3	2.9430	49.3019	2.75	2	0.8563	39.3675
4.5	0	∞	50.0000	3.0	0	0	38.3333
t	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$	ε	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
1.0	2	1.6063	45.8134	0.25	2	1.6063	44.8166
1.25	2	1.6063	46.0458	0.50	2	1.6063	45.7567
1.5	1	0.9368	45.8634	0.75	2	1.6063	47.9211
2.0	2	1.6063	44.8481	1.00	2	1.6063	44.8481
2.5	2	1.6063	43.7081	1.50	3	2.2749	44.8428
3.0	2	1.6063	43.2756	1.75	3	2.2749	45.0192
4.0	2	1.6063	42.7466	2.00	3	2.2749	46.0138
a_0	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$	a_1	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
0	5	2.2158	32.4155	0	2	1.1623	41.9663
10	2	1.0689	39.8145	1	2	1.2467	43.0501
15	2	1.3066	41.7156	3	2	1.4221	43.8116
20	2	1.6063	44.8481	5	2	1.6063	44.8481
25	3	2.7425	47.1352	7	3	2.5066	46.0174
30	3	3.3935	48.3154	10	3	2.8730	46.3247
35	0	∞	50.0000	15	3	3.5311	48.5122
a_2	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$	C_1	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
4	0	0.00	39.0000	0.1	3	2.2749	41.5121
6	5	2.5195	41.3122	0.2	3	2.2749	41.8121
8	2	1.2756	43.7820	0.4	3	2.2749	42.4121
10	2	1.6063	44.8481	0.5	2	1.6063	44.8481
12	3	2.6292	46.1724	0.6	2	1.6063	45.0481
15	3	3.1098	47.0937	0.8	2	1.6063	45.4481
20	1	2.0000	48.5000	1.0	2	1.6063	45.8491
C_2	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$	C_3	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
0.1	3	2.2749	42.9141	35	0	∞	35.0000
0.2	3	2.2749	43.1654	40	3	3.3935	38.3154
0.4	3	2.2749	43.8762	45	3	2.7425	42.2593
0.5	2	1.6063	44.8481	50	2	1.6063	44.5118
0.6	3	1.6063	45.3412	55	2	1.3066	46.7166
0.8	2	0.9368	45.5788	60	2	1.0689	49.9954
1.0	2	2.6292	46.1455	70	5	2.2158	52.3851

Table 2
Under t-FCT, optimal solutions $(n_{B_2}, t_{B_2}, \delta_{B_2})$ and its Bayes risks

α	n_{B_2}	t_{B_2}	$D_n^*(m)$	$r(n_{B_2}, \delta_{B_2})$	β	n_{B_2}	t_{B_2}	$D_n^*(m)$	$r(n_{B_2}, \delta_{B_2})$
1.5	1	1.8508	0.0712	31.6250	1.0	0	1	∞	50.0000
2.0	2	1.6225	0.2656	37.2154	1.25	1	1.0125	1.6868	47.8006
2.5	2	1.2658	0.6014	41.3250	1.5	3	1.4025	1.4368	46.5411
3.0	2	1.6758	0.9368	44.3211	2.0	2	1.6758	0.9368	44.3211
3.5	2	1.9775	1.2717	45.7145	2.5	2	1.6775	0.4368	42.3185
4.0	2	1.6742	1.6063	48.2145	2.75	2	1.4525	0.1838	38.8169
4.5	0	2.0000	∞	50.0000	3.0	0	2.0000	0.0000	38.3333
a_0	n_{B_2}	t_{B_2}	$D_n^*(m)$	$r(n_{B_2}, \delta_{B_2})$	a_1	n_{B_2}	t_{B_2}	$D_n^*(m)$	$r(n_{B_2}, \delta_{B_2})$
0	4	1.8258	0.2100	31.8214	0	2	1.6475	0.5820	41.0217
10	2	1.6225	0.5000	39.0172	1	2	1.8258	0.6495	43.5145
15	2	1.6608	0.6932	41.4187	3	1	1.4092	0.7897	43.1167
20	2	1.6758	0.9368	44.3211	5	2	1.6758	0.9368	44.3211
25	3	1.9758	1.2566	46.1342	7	1	1.6475	1.0905	46.0123
30	3	1.6225	1.7016	47.7451	10	1	1.6042	1.3333	46.6012
35	0	2.0000	∞	50.0000	15	1	1.4358	1.7689	48.0122
a_3	n_{B_2}	t_{B_2}	$D_n^*(m)$	$r(n_{B_2}, \delta_{B_2})$	C_1	n_{B_2}	t_{B_2}	$D_n^*(m)$	$r(n_{B_2}, \delta_{B_2})$
4	0	2.0000	0.0000	39.0000	0.1	1	1.8242	1.6063	41.7052
6	1	1.5675	0.3609	42.0124	0.2	1	1.8242	1.6063	41.8752
8	1	1.8258	0.6667	43.9991	0.4	1	1.8242	1.6063	42.0752
10	2	1.6758	0.9368	44.3211	0.5	2	1.6758	0.9368	44.3211
12	1	1.4042	1.1813	46.2417	0.6	2	1.6758	0.9368	44.5211
15	1	1.5325	1.5131	47.2915	0.8	2	1.6758	0.9368	44.9211
20	1	1.6275	2.0000	48.0122	1.0	2	1.6758	0.9368	45.3211
C_2	n_{B_2}	t_{B_2}	$D_n^*(m)$	$r(n_{B_2}, \delta_{B_2})$	C_3	n_{B_2}	t_{B_2}	$D_n^*(m)$	$r(n_{B_2}, \delta_{B_2})$
0.1	3	1.5675	1.6063	42.3151	35	0	2.0000	∞	35.0000
0.2	3	1.8242	1.6063	42.8023	40	3	1.8425	1.7016	37.9142
0.4	3	1.3258	1.6063	43.6112	45	3	1.6525	1.2566	40.5121
0.5	2	1.6758	0.9368	44.3211	50	2	1.6758	0.9368	44.3211
0.6	2	1.4058	0.9368	44.7152	55	2	1.4092	0.6932	46.4451
0.8	2	1.9792	0.9368	45.0123	60	2	1.8592	0.5000	49.4132
1.0	2	1.5442	0.9368	45.5214	70	5	1.9942	0.2100	51.7812

employed by many statisticians. Wetherill and Campling (1966) and Köllerström and Wetherill (1981) applied this approach and considered the utility function for sampling plans by attributes as well as by variables. Fertig and Mann (1974), Hald (1967, 1981) and Wetherill and Köllerström (1979) investigated the asymptotic results of the sampling plans. However, in these papers, they dealt with linear loss function and so the sample size obtained by the proposed optimal sampling plan was usually not an integer. Lam (1988a, b, 1994) and Lam and Lau (1993) developed some models and studied some optimal sampling plans for polynomial loss and derived explicit forms of

Table 3
Under ε-FCT, optimal solutions $(n_{B_3}, \varepsilon_{B_3}, \delta_{B_3})$ and its Bayes risks

α	n_{B_3}	ε_{B_3}	$D_n^*(m)$	$r(n_{B_3}, \delta_{B_3})$	β	n_{B_3}	ε_{B_3}	$D_n^*(m)$	$r(n_{B_3}, \delta_{B_3})$
1.5	3	0.8122	0.0712	30.8142	1.0	0	1.0000	∞	50.0000
2.0	2	0.4158	0.2656	36.8241	1.25	1	0.1595	1.6868	48.7084
2.5	2	0.3266	0.6014	40.7512	1.5	3	0.3278	2.7749	48.3723
3.0	2	0.2500	0.9368	44.3012	2.0	2	0.2500	0.9368	44.3012
3.5	2	0.4266	1.2717	44.6145	2.5	2	0.5125	1.1063	43.4753
4.0	3	0.3311	1.6063	47.7021	2.75	2	0.4454	0.8563	39.3154
4.5	0	1.0000	∞	50.0000	3.0	0	1.0000	0.0000	38.3333
a_0	n_{B_3}	ε_{B_3}	$D_n^*(m)$	$r(n_{B_3}, \delta_{B_3})$	a_1	n_{B_3}	ε_{B_3}	$D_n^*(m)$	$r(n_{B_3}, \delta_{B_3})$
0	4	0.3215	0.2100	30.9549	0	2	0.1648	1.1623	41.0122
10	2	0.4215	0.5000	37.8145	1	2	0.1587	1.2467	43.5014
15	2	0.1455	0.6932	39.7168	3	2	0.3214	1.4221	42.5725
20	2	0.2500	0.9368	44.3012	5	2	0.2500	0.9368	44.3012
25	3	0.2215	1.2566	44.7544	7	3	0.5416	2.5066	45.8525
30	3	0.3248	1.7016	46.1067	10	3	0.3123	2.8730	46.2871
35	0	1.0000	∞	50.0000	15	3	0.3225	3.5311	47.7141
a_2	n_{B_3}	ε_{B_3}	$D_n^*(m)$	$r(n_{B_3}, \delta_{B_3})$	C_1	n_{B_3}	ε_{B_3}	$D_n^*(m)$	$r(n_{B_3}, \delta_{B_3})$
4	0	0.1655	0.0000	39.0000	0.1	3	0.4015	1.1813	40.8854
6	3	0.3845	0.3609	41.6511	0.2	3	0.1254	1.1813	41.1854
8	2	0.2154	0.6667	43.5674	0.4	3	0.2255	1.1813	41.7854
10	2	0.2500	0.9368	44.3012	0.5	2	0.2500	0.9368	44.3012
12	3	0.4152	1.1813	45.8121	0.6	2	0.2415	0.9368	44.5012
15	3	0.3211	1.5131	47.0011	0.8	2	0.4332	0.9368	44.9012
20	1	0.5105	2.0000	47.6152	1.0	2	0.2155	0.9368	45.3012
C_2	n_{B_3}	ε_{B_3}	$D_n^*(m)$	$r(n_{B_3}, \delta_{B_3})$	C_3	n_{B_3}	ε_{B_3}	$D_n^*(m)$	$r(n_{B_3}, \delta_{B_3})$
0.1	3	0.3124	1.1813	42.5123	35	0	1.0000	∞	35.0000
0.2	3	0.1455	1.1813	42.6521	40	3	0.3124	1.7016	37.7121
0.4	3	0.3211	1.1813	43.3215	45	3	0.4158	1.2566	41.0120
0.5	2	0.2500	0.9368	44.3012	50	2	0.2500	0.9368	44.3012
0.6	2	0.2144	1.6063	44.6241	55	2	0.4011	0.6932	46.8158
0.8	2	0.2255	1.2717	45.0003	60	2	0.2175	0.5000	49.8514
1.0	2	0.4215	1.7016	45.4192	70	5	0.1256	0.2100	51.6074

the Bayes risks. Therefore, an optimal plan with an integer-valued sample size can be obtained within finite-step of searching.

In testing lifetimes of electronics or testing survival times of patients who suffer from serious diseases, measurements are usually censored. Usually, there are three kinds of censoring. Type II censoring is generally used when items in a large batch are sophisticated and/or expensive. In this case, inspection terminates when a pre-assigned number of defective items have been found in a fixed size sample. However, Type I censoring is employed if the inspection cost increases heavily with time. Life times

Table 4
Under FUCT, optimal solutions $(n_{B_4}, t_{B_4}, \varepsilon_{B_4}, \delta_{B_4})$ and its Bayes risks

α	$n_{B_4 n}$	t_{B_4}	ε_{B_4}	$D_n^*(m)$	$r(n_{B_4}, \delta_{B_4})$	β	n_{B_4}	t_{B_4}	ε_{B_4}	$D_n^*(m)$	$r(n_{B_4}, \delta_{B_4})$
1.5	1	1.8592	0.2512	0.0712	30.3022	1.0	0	2.0000	1.0000	∞	50.0000
2.0	2	1.6225	0.4545	0.2656	35.3618	1.25	1	1.5242	0.1255	2.3563	45.1243
2.5	2	1.5675	0.1244	0.6014	39.3214	1.5	3	1.6225	0.4585	2.1063	44.5124
3.0	2	1.6642	0.2452	1.6063	43.8715	2.0	2	1.6642	0.2452	1.6063	43.8715
3.5	2	1.9758	0.4518	1.2717	43.5541	2.5	2	1.4042	0.4031	0.4368	40.0614
4.0	2	1.8258	0.3213	1.6063	47.2132	2.75	2	1.1258	0.1175	0.1868	38.4152
4.5	0	2.0000	1.0000	∞	50.0000	3.0	0	2.0000	1.0000	0.0000	38.3333

a_0	n_{B_4}	t_{B_4}	ε_{B_4}	$D_n^*(m)$	$r(n_{B_4}, \delta_{B_4})$	a_1	n_{B_4}	t_{B_4}	ε_{B_4}	$D_n^*(m)$	$r(n_{B_4}, \delta_{B_4})$
0	2	1.5358	0.4215	0.7122	30.0685	0	2	1.9758	0.2515	1.1623	40.0121
10	2	1.8258	0.2578	1.0689	37.7214	1	2	1.6742	0.1145	1.2467	41.9452
15	2	1.7025	0.0125	1.6066	40.3965	3	2	1.9875	0.4453	1.4221	41.6701
20	2	1.6642	0.2452	1.6063	43.8715	5	2	1.6642	0.2452	1.6063	44.1308
25	2	1.7458	0.7127	2.0000	44.8451	7	2	1.7008	0.3325	1.7990	44.8451
30	2	1.6225	0.3452	2.5481	47.6452	10	2	1.6708	0.0250	2.1036	46.5141
35	0	2.0000	1.0000	∞	50.0000	15	2	1.8525	0.1783	2.6504	47.5142

a_2	n_{B_4}	t_{B_4}	ε_{B_4}	$D_n^*(m)$	$r(n_{B_4}, \delta_{B_4})$	C_1	n_{B_4}	t_{B_4}	ε_{B_4}	$D_n^*(m)$	$r(n_{B_4}, \delta_{B_4})$
4	0	2.0000	1.0000	0.0000	39.0000	0.1	2	1.5125	0.2245	1.2717	40.6128
6	2	1.6575	0.3323	0.9013	41.1421	0.2	2	1.5125	0.2245	1.2717	40.8128
8	2	1.6025	0.2038	1.2756	43.1384	0.4	2	1.5125	0.2245	1.2717	41.2128
10	2	1.6642	0.2452	1.6063	43.8715	0.5	2	1.6642	0.2452	1.6063	43.8715
12	2	1.9242	0.4213	1.9057	45.6877	0.6	2	1.6642	0.2452	1.6063	44.0715
15	2	1.7375	0.2150	2.3120	46.8515	0.8	2	1.6642	0.2452	1.6063	44.4715
20	2	1.9025	0.3867	2.9082	47.3123	1.0	2	1.6642	0.2452	1.6063	44.8715

C_2	n_{B_4}	t_{B_4}	ε_{B_4}	$D_n^*(m)$	$r(n_{B_4}, \delta_{B_4})$	C_3	n_{B_4}	t_{B_4}	ε_{B_4}	$D_n^*(m)$	$r(n_{B_4}, \delta_{B_4})$
0.1	2	1.6025	0.4257	1.6063	42.3612	35	0	2.0000	1.0000	∞	35.0000
0.2	2	1.8508	0.3125	1.6063	42.5545	40	2	1.4542	0.3242	2.5481	37.5052
0.4	2	1.4225	0.1122	1.6063	43.0134	45	2	1.7275	0.2144	2.0000	39.8714
0.5	2	1.6642	0.2452	1.6063	43.8715	50	2	1.6642	0.2452	1.6063	43.8715
0.6	2	1.6325	0.1158	1.6063	44.5128	55	2	1.5875	0.4215	1.3066	46.2154
0.8	2	1.8525	0.3245	1.6063	44.5712	60	2	1.8875	0.2565	1.0689	48.6542
1.0	2	1.6292	0.2454	1.6063	44.9019	70	2	1.6725	0.1187	0.7122	51.2566

of items in the sample survive beyond a pre-assigned time t are censored. For other situations, the lifetime of an item may simultaneously be affected by an extraneous factor. For example, a group of patients who suffer from both disease A and disease B are given a new drug which is claimed to be a new treatment for disease A. The lifetime of each participating patient is reported and recorded. For each patient, a censoring time is assigned which corresponds to the survival time of disease B for the patient. These censoring times are assumed to be identically and independently distributed (IID) with known distribution and hence for this case random censoring is used.

Table 5
Under FCT, optimal solutions (n_0, T_0) and its Bayes risks of Lam and Choy (1995) and (n_{B_1}, δ_{B_1}) with $C_2 = 0$

α	n_0	T_0	$r(n_0, T_0)$	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
1.5	0	0.0000	33.1250	4	1.9407	31.2416
2.0	3	0.5400	38.3390	2	0.9368	36.5095
2.5	4	0.6600	42.5279	2	1.2717	41.1977
3.0	4	0.7400	45.8851	2	1.6063	44.0148
3.5	3	0.9200	48.3002	2	1.9407	45.9362
4.0	1	1.6000	49.7467	3	2.9430	48.4269
4.5	0	∞	50.0000	0	∞	50.0000
β	n_0	T_0	$r(n_0, T_0)$	n_{B_1}	$D_n^*(m)$	$r(n_{B_1}, \delta_{B_1})$
1.00	0	∞	50.0000	0	∞	50.0000
1.25	1	1.6800	49.8150	1	1.6868	47.6654
1.50	3	1.0200	48.8.95	3	2.7749	47.4923
2.00	4	0.7400	45.8851	2	1.6063	44.0148
2.50	4	0.6800	44.1469	2	1.1063	42.3400
2.75	3	0.5000	40.5323	2	0.8563	38.5342
3.00	0	0	38.3333	0	0.0000	38.3333

In this paper, our goal is to seek an optimal sampling plan (n_B, δ_B) possessing the property that the risk $r(n_B, \delta_B) = \inf r(n, \delta)$ among the class of all sampling plans (n, δ) based on data which are uniformly random censored. We set up a decision-theoretic formulation of the problem of acceptance sampling in Section 2. A Bayesian sampling plan is derived. In Section 3, a special case where $h(\lambda) = a_0 + a_1\lambda + a_2\lambda^2$ is considered. We provide an explicit presentation of the Bayes risk of a sampling plan $r(n, \delta_B(\,|\,n))$. Based on this expression, a numerical approximation for finding the optimal sample size n_B and the optimal decision rule is proposed. In Section 4, an algorithm for determining the optimal sampling plan (n_B, δ_B) is given and some numerical results related to optimal sampling plans are tabulated (Tables 1–4). Under special loss function, some numerical comparisons of Bayes risks between the proposed optimal sampling plan and that of Lam and Choy (1995) are also studied (Table 5).

2. Formulation of the model and a Bayes solution

Let X denote the lifetime of an item in a batch of size N. Assume that X has an exponential distribution Exp(λ) with density function $f(x|\lambda) = \lambda e^{-\lambda x}$ for $x > 0$ and 0 otherwise. Here the scale parameter λ is unknown; however, we assume it follows a conjugate Gamma prior distribution $\Gamma(\alpha, \beta)$ with density function $g(\lambda) = \beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda}/\Gamma(\alpha)$ for $\lambda > 0$ and 0 otherwise, where α and β are known.

In designing a sampling scheme, a random sample $\mathbf{X} = (X_1, \ldots X_n)$ of fixed size n is taken from the batch for testing. Assume that random censoring is adopted. Let the censoring times $Y_1, \ldots, Y_n$ be IID random variables associated with the true lifetime

$X_1,\ldots,X_n$, respectively. Suppose the Y_i's and X_i's are independent and Y_i $(i=1,\ldots,n)$ is uniformly distributed over an interval $[t-\varepsilon, t]$ with known $t \geqslant \varepsilon > 0$. This kind of random censoring is commonly used in practical applications. As Ebrahimi and Habibullah (1992) have pointed out, in most clinical trials, with staggered entries over an initial period $(0,\varepsilon)$ for accrual, and with analysis at some time $t > \varepsilon$, it is realistic to assume that the censoring time Y_i associated with X_i is uniformly distributed on $[t-\varepsilon, t]$.

Following the usual notation, the observable data are given by the pair (Z_i, δ_i), $i = 1,\ldots,n$, where

$$Z_i = \min(X_i, Y_i) = X_i \wedge Y_i$$

and

$$\delta_i = I_{(X_i \leqslant Y_i)} = \begin{cases} 1 & \text{if } X_i \leqslant Y_i, \\ 0 & \text{if } X_i > Y_i. \end{cases} \tag{2.1}$$

It is readily seen that when $\varepsilon \to 0$, the uniform random censoring becomes the usual Type I censoring. Accordingly, the model under consideration is an extension of that of the Type I censoring model.

Let M denote the number of failures by time t, i.e. $M = \sum_{i=1}^{n} \delta_i$, then M follows a binomial distribution $B(n, p)$ where

$$p = \Pr(X_i \leqslant Y_i|\lambda) = 1 - \frac{1}{\lambda\varepsilon}\{\exp[-\lambda(t-\varepsilon)] - \exp(-\lambda t)\}.$$

For the fixed sample size n and the censoring time t, given $M = m$, let $\underset{\sim}{Z}(N) = (Z_1, 1, Z_2, 1, \ldots, Z_m, 1, Z_{m+1}, 0, \ldots, Z_n, 0)$ be the observable lifetimes of the M failed components. Then, given λ, the joint density function of $(\underset{\sim}{Z}(N), M)$ is given by

$$f(z_1, 1, \ldots, z_m, 1, z_{m+1}, 0, \ldots, z_n, 0; m|\lambda)$$
$$= \begin{cases} \begin{pmatrix} n \\ m \end{pmatrix} \prod_{i=1}^{n} f(z_i, \delta_i) & 0 \leqslant z_i \leqslant t, \text{ for } i = 1, \ldots, m, \text{ or} \\ & t-\varepsilon \leqslant z_i \leqslant t, \; i = m+1, \ldots, n, \\ 0 & \text{otherwise,} \end{cases} \tag{2.2}$$

where

$$f(z, \delta) = \begin{cases} \lambda \exp(-\lambda z), & 0 \leqslant z < t-\varepsilon, \; \delta = 1, \\ \exp(-\lambda z)/\varepsilon, & t-\varepsilon \leqslant z \leqslant t, \; \delta = 0, \\ \lambda \exp(-\lambda z)(t-z)/\varepsilon, & t-\varepsilon \leqslant z \leqslant t, \; \delta = 1. \end{cases} \tag{2.3}$$

Therefore, (2.2) becomes

$$f(z_1, 1, \ldots, z_m, 1, z_{m+1}, 0, \ldots, z_n, 0; m|\lambda)$$
$$= \begin{cases} \begin{pmatrix} n \\ m \end{pmatrix} \lambda^m \exp(-\lambda z(n)) & \\ \prod_{i=1}^{m} \left\{ \frac{t - z_i}{\varepsilon} \wedge 1 \right\} \left(\frac{1}{\varepsilon} \right)^{n-m} & 0 \leqslant z_i \leqslant t, \text{ for } i = 1, \ldots, m, \text{ or} \\ & t - \varepsilon \leqslant z_i \leqslant t, \ i = m+1, \ldots, n, \\ 0 & \text{otherwise,} \end{cases} \tag{2.4}$$

where $z(n) = \sum_{i=1}^{n} z_i$, $(n-m)(t-\varepsilon) \leqslant \sum_{i=1}^{n} z_i \leqslant nt$.

Note that $(z_1, 1, \ldots, z_m, 1, z_{m+1}, 0, \ldots, z_n, 0, M)$ are sufficient for λ. It is assumed that the parameter λ is a realization of a positive random variable Λ, having a prior density $g(\lambda)$ over $(0, \infty)$. Therefore, the marginal joint probability density function of $(\underset{\sim}{Z}(N), M)$ is

$$f(z_1, 1, \ldots, z_m, 1, z_{m+1}, 0, \ldots, z_n, 0, m)$$
$$= \int_0^\infty f(z_1, 1, \ldots, z_m, 1, z_{m+1}, 0, \ldots, z_n, 0, m|\lambda) g(\lambda)\, \mathrm{d}\lambda$$
$$= \begin{pmatrix} n \\ m \end{pmatrix} \int_0^\infty \lambda^m \exp\left(-\lambda \sum_{i=1}^{i} z_i \right) \prod_{i=1}^{m} \left\{ \frac{t - z_i}{\varepsilon} \wedge 1 \right\} \left(\frac{1}{\varepsilon} \right)^{n-m} g(\lambda)\, \mathrm{d}\lambda. \tag{2.5}$$

The posterior probability density of λ given $(\underset{\sim}{Z}(N), M) = (\underset{\sim}{z}(n), m)$ is then given by

$$g(\lambda|\underset{\sim}{z}(n), m) = \frac{f(\underset{\sim}{z}(n), m \mid \lambda) g(\lambda)}{f(\underset{\sim}{z}(n), m)}$$
$$= \frac{\lambda^m \exp\left\{ -\lambda \sum_{i=1}^{n} z_i \right\} g(\lambda)}{\int_0^\infty \lambda^m \exp\left\{ -\lambda \sum_{i=1}^{n} z_i \right\} g(\lambda)\, \mathrm{d}\lambda}. \tag{2.6}$$

Lam and Choy (1995) have considered the same problem through a Bayesian setup. They derived a Bayesian sampling plan based on a special decision function which is defined in terms of the MLE $\hat{\theta}$ of θ. Theoretically, since it has not been confirmed that the associated risk attains the infimum over all reasonable decision functions, their derived optimal plan is not assured to be a real Bayesian solution. As a matter of fact, their optimal solution is not Bayes which can be confirmed by (3.3) in the next section. This is also shown by some numerical comparisons of risks tabulated in Table 5 in Section 4.

In many life testing situations or clinical trials, it often takes a long time to observe complete life times. This is quite undesirable or even impossible due to various restrictions on the experiment, for instance, budget restrictions. Therefore, it is desirable to have the experiment terminated as soon as the accumulated data is sufficient for our goal. In this sense, the censoring time Y_i can be designed according to some criterion. We consider four situations for the design of Y_i in our paper.

In some situation, due to some constraints or requirements, the parameters t and ε in uniform distribution $\mathrm{U}(t-\varepsilon,t)$ are both fixed. We call this situation the fixed censoring time (FCT). This case has been studied in Lam and Choy (1995). For the second situation, the parameter ε is fixed, however, another parameter t is allowed to be chosen case by case for the benefit of some purpose. For this model, we call it the t-flexible censoring time (t-FCT). On the other hand, in some situation, the parameter t is fixed and ε is allowed to be flexible. As is well-known, when ε is restricted to be small, this censoring model is close to Type I censoring. For this case it is called the ε-flexible censoring time (ε-FCT).

Finally, when the experiment is very flexible in determining its censoring time, it is permitted that both t and ε can be chosen by experimenter before the experiment starts. For this censoring scheme, we call it a flexible uniform censoring time (FUCT).

In this paper, we derive the Bayesian sampling plan under various situations of censoring time. Obviously, for the cases of t-FCT, ε-FCT and FUCT, they are not studied in Lam and Choy (1995). In the problem formulation, we consider an important factor of time in our loss function. Under this situation, the censoring schemes t-FCT and FUCT are rather significant and important in the sampling plans.

Suppose that a batch of lifetime components is presented for acceptance sampling. Let a denote an action on this problem of acceptance sampling. When $a=1$, it means that the batch is accepted; and when $a=0$, the batch is to be rejected. For given sample size n, censoring time $\underset{\sim}{Y}=(Y_1,Y_2,\ldots,Y_n)$ and parameter λ, when action a is taken, the loss is defined as follows:

$$L(a,\lambda,n)=ah(\lambda)+(1-a)C_3+nC_1+\max_{1\leqslant i\leqslant n}Y_iC_2, \tag{2.7}$$

where C_1, C_2 and C_3 are all positive constants, and they denote, respectively, the cost per item inspected, the cost per unit time used for test and the loss due to rejecting the batch, and $h(\lambda)$ denotes the loss of accepting the batch. Since $\theta=\lambda^{-1}$ is the expected lifetime, and a larger λ indicates a smaller θ, so, usually, we require $h(\lambda)$ to be positive and increasing in λ for $\lambda>0$. Also, to ensure the Bayes risk to be finite, it is assumed that $\int_0^\infty h(\lambda)g(\lambda)\,\mathrm{d}\lambda<\infty$.

It should be emphasized that the cost C_2 for unit time in loss $L(a,\lambda,n)$ is an important term to be considered since it is closely related to random censoring scheme and thus it controls the total length of time of items inspection. Due to budget restrictions or some constraint on the experiment, practically it is necessary to consider cost of time.

Using the loss $L(a,\lambda,n)$ and applying some conditioning technique, the Bayes risk of a sampling plan (n,δ) can be computed and decomposed in the following form:

$$
\begin{aligned}
r(n,\delta) &= E_{\underset{\sim}{Y}}\left(\max_{1\leqslant i\leqslant n} Y_i\right) C_2 \\
&\quad + E_\Lambda E_{\underset{\sim}{Z}(N),M|\Lambda}\{nC_1 + C_3 + \delta(\underset{\sim}{Z}(N),M|n)[h(\Lambda) - C_3]\} \\
&= \left(t - \frac{\varepsilon}{n+1}\right) C_2 + nC_1 + C_3 + r_1(\delta|n),
\end{aligned} \tag{2.8}
$$

where

$$
\begin{aligned}
r_1(\delta\mid n) &= E_\Lambda E_{\underset{\sim}{Z}(N),M\mid\Lambda}\{\delta(\underset{\sim}{Z}(N),M\mid n)[h(\Lambda)-C_3]\} \\
&= E_{\underset{\sim}{Z}(N),M} E_{\Lambda|\underset{\sim}{Z}(N),M}\{\delta(\underset{\sim}{Z}(N),M\mid n)[h(\Lambda)-C_3]\} \\
&= \sum_{m=0}^{n} \int \underset{\underset{\sim}{z}(n)}{\cdots} \int \delta(\underset{\sim}{z}(n),m\mid n)\{E_{\Lambda|\underset{\sim}{z}(n),m}[h(\Lambda)-C_3]\} \\
&\quad f(\underset{\sim}{z}(n),m)\,\mathrm{d}\underset{\sim}{z}(n)
\end{aligned} \tag{2.9}
$$

and

$$
\begin{aligned}
E_{\Lambda|\underset{\sim}{z}(n),m}[h(\Lambda)-C_3] &= \int_0^\infty h(\lambda)g(\lambda\mid \underset{\sim}{z}(n),m)\ \mathrm{d}\lambda - C_3 \\
&= \varphi_g(z(n),m) - C_3,
\end{aligned} \tag{2.10}
$$

where $\varphi_g(z(n),m) = \int_0^\infty h(\lambda)g(\lambda\mid \underset{\sim}{z}(n),m)\,\mathrm{d}\lambda$, the posterior expectation of $h(\Lambda)$ given $(\underset{\sim}{Z}(N),M) = (\underset{\sim}{z}(n),m)$.

Therefore, for a fixed sample size n, given parameters t and ε in uniform censoring, the Bayes decision function $\delta_{\mathrm{B}}(\mid n)$, which minimizes $r_1(\delta\mid n)$ among all decision functions $\delta(\mid n)$ is given by

$$
\delta_{\mathrm{B}}(\underset{\sim}{z}(n),m|n) = \begin{cases} 1 & \text{if } \varphi_g(z(n),m) \leqslant C_3, \\ 0 & \text{otherwise.} \end{cases} \tag{2.11}
$$

Next, we investigate some monotonicity properties of the Bayes decision function $\delta_{\mathrm{B}}(\mid n)$ with n fixed. Main property of $\delta_{\mathrm{B}}(\cdot)$ defined by (2.11) is given by (b) of the following Theorem 2.1.

Lemma 2.1. *Let $0 \leqslant m^*, m \leqslant n$ and $z = z(n)$ associated with m defined by (2.4), $z^* = z(n)$ associated with m^*. Consider the likelihood ratio*

$$
\ell(\lambda|(z,m),(z^*,m^*)) = g(\lambda|z^*,m^*)/g(\lambda|z,m) \quad \text{if } g(\lambda|z,m) \neq 0.
$$

The following holds.

(a) *If $m = m^*$ and $z < z^*$, then $\ell(\lambda|(z,m),(z^*m^*))$ is nonincreasing in λ.*
(b) *If $z = z^*$ and $m < m^*$, then $\ell(\lambda|(z,m),(z^*m^*))$ is nondecreasing in λ.*

Proof. The proof is straightforward, so we omit the proof. □

From the above fact, we can conclude the following result.

Theorem 2.1. *Let $h(\lambda)$ be a positive and increasing function of λ for $\lambda > 0$. Then,*

(a) $\varphi_g(z,m) = \int_0^\infty h(\lambda)g(\lambda|z,m)\,\mathrm{d}\lambda$ *is nonincreasing in z and nondecreasing in m.*
(b) $\delta_{\mathrm{B}}(\underset{\sim}{z}(n),m|n)$ *is nondecreasing in $z(n)$ and nonincreasing in m.*

Proof. It follows from Lemma 2.1 that the conditional density $g(\lambda;z_1,m)$ is a family of densities with monotone likelihood ratio in λ considering m as a parameter. Then, if $h(\lambda)$ is nondecreasing in λ, $E_\lambda h(\lambda) = \varphi_g(z,m)$ is nondecreasing in m (see, for example Lehmann, 1959, p. 74). The same method can be used to show that $\varphi_g(z,m)$ is also nonincreasing in z. This shows (a).

(b) follows directly from (a) using (2.11).

For fixed n, since $\delta_{\mathrm{B}}(z(n),m|n)$ is nondecreasing in $z(n)$, it is to be noted that a bigger value of $z(n)$ leads to a bigger value of $\delta(\cdot)$ and thus it results a bigger probability for accepting the batch.

2.1. Derivation of a Bayesian sampling plan

To derive a Bayesian sampling plan under various situations, the following schemes are proposed.

(A) *Both t and ε are prefixed* (*FCT*)

Scheme A1.

Step 1: For fixed n, derive the decision function $\delta_{\mathrm{B}_1}(n)$, which minimizes $r_1(\delta_{\mathrm{B}_1}|n)$ (defined by (2.10) and (2.11)) among all the decision function δ. So, $\delta_{\mathrm{B}_1}(n)$ satisfies $r_1(\delta_{\mathrm{B}_1}|n) = \inf\{r_1(\delta|n)\}$.

Step 2: Find the sample size n_{B_1} which minimizes $r(n,\delta_{\mathrm{B}_1}(|n))$ (defined by (2.9)) among all $n = 0,1,2,\dots$.

Then, $(n_{\mathrm{B}_1},\delta_{\mathrm{B}_1})$ is our Bayes solution.

(B) *ε is prefixed and t is flexible* (*t-FCT*)

Scheme A2.

Step 1: For fixed (n,t), derive the decision function $\delta_{\mathrm{B}_2}(\,|n)$ to minimize the risks $r_1(\delta|n)$ among all decision functions $\delta(\,|n)$.

Step 2: For fixed n, derive the censoring time $t_{\mathrm{B}_2}(n)$, which minimizes $(t-\varepsilon/(n+1))C_2 + r_1(\delta_{\mathrm{B}_2}|n)$ among $t>0$. That is, $t_{\mathrm{B}_2}(n)$ satisfies $(t_{\mathrm{B}_2}(n)-\varepsilon/(n+1))\,C_2 + r_1(\delta_{\mathrm{B}_2}|n) = \inf_{t\geqslant\varepsilon}\{(t-\varepsilon/(n+1))\,C_2 + r_1(\delta_{\mathrm{B}_2}|n)\}$.

Step 3: Find the sample size n_{B_2} which minimizes $r(n,\delta_{\mathrm{B}_2}(|n))$ among all $n = 0,1,2,\dots$.

Then, $(n_{\mathrm{B}_2},t_{\mathrm{B}_2}(n_{\mathrm{B}_2}),\delta_{\mathrm{B}_2})$ is our Bayes solution.

(C) *t is prefixed and ε is flexible* (*ε-FCT*)

Scheme A3.

Step 1: For fixed (n,ε), derive the decision function $\delta_{\mathrm{B}_3}(\,|n)$ to minimize the risks $r_1(\delta|n)$ among all decision functions $\delta(\,|n)$.

Step 2: For fixed n, derive $\varepsilon_{B_3}(n)$, which minimizes $(t-\varepsilon/(n+1))C_2+r_1(\delta_{B_3}(\ |n))$ among $t \geqslant \varepsilon > 0$. That is, $\varepsilon_{B_3}(n)$ satisfies $(t-\varepsilon_{B_3}(n)/(n+1))C_2+r_1(\delta_{B_3}|n)=\inf_{0<\varepsilon\leqslant t}\{(t-\varepsilon/(n+1))C_2+r_1(\delta_{B_3}|n)\}$.

Step 3: Find the sample size n_{B_3} which minimizes $r(n,\delta_{B_3}(\ |n))$ among all $n=0,1,2,\ldots$.

So, $(n_{B_3},\varepsilon_{B_3}(n_{B_3}),\delta_{B_3})$ is our Bayes solution.

(D) *Both t and ε are flexible* (*FUCT*)

Scheme A4.

Step 1: For fixed (n,t,ε), derive the decision function $\delta_{B_4}(\ |n)$ to minimize the risks $r_1(\delta|n)$ among all decision functions $\delta(\ |n)$.

Step 2: For fixed n, derive $t_{B_4}(n)$ and $\varepsilon_{B_4}(n)$ $(0<\varepsilon_{B_4}(n)\leqslant t_{B_4}(n))$ which minimize $(t-\varepsilon/(n+1))C_2+r_1(\delta_B|n)$ among $t\geqslant\varepsilon>0$. That is, $t_{B_4}(n)$ and $\varepsilon_{B_4}(n)$ satisfy $(t_{B_4}(n)-1/(n+1)\varepsilon_{B_4}(n))C_2+r_1(\delta_{B_4}|n)=\inf_{0<\varepsilon\leqslant t}\{(t-\varepsilon/(n+1))C_2+r_1(\delta_{B_4}|n)\}$.

Step 3: Find the sample size n_{B_4} which minimizes $r(n,\delta_{B_4}(|n))$ among all $n=0,1,2,\ldots$.

Then, $(n_{B_4},t_{B_4}(n_{B_4}),\varepsilon_{B_4}(n_{B_4}),\delta_{B_4})$ is our Bayes solution.

All the sampling plans derived through the Schemes A1–A4, respectively, possess the following optimality property.

Theorem 2.2. Sampling plans (n_{B_1},δ_{B_1}) for the case FCT, $(n_{B_2},t_{B_2}(n_{B_2}),\delta_{B_2})$ for t-FCT, $(n_{B_3},\varepsilon_{B_3}(n_{B_3}),\delta_{B_3})$ for ε-FCT and $(n_{B_4},t_{B_4}(n_{B_4}),\varepsilon_{B_4}(n_{B_4}),\delta_{B_4})$ for FUCT are Bayes sampling plans in the sense that each of them attains $\inf r(n,\delta)$ among the class of all sampling plans for each situation.

Proof. Since proof for each situation in analogous, we consider here the case of FUCT.

For any sampling plan (n,t,ε,δ), we use $r(n,t,\varepsilon,\delta)$ to denote its risk. Then, we have

$$\begin{aligned} &r(n,t,\varepsilon,\delta)-r(n_{B_4},t_{B_4}(n_{B_4}),\varepsilon_{B_4}(n_{B_4}),\delta_{B_4})\\ &\quad=r(n,t,\varepsilon,\delta)-r(n,t,\varepsilon,\delta_{B_4})+r(n,t,\varepsilon,\delta_{B_4})-r(n,t_{B_4}(n),\varepsilon_{B_4}(n),\delta_{B_4})\\ &\qquad+r(n,t_{B_4}(n),\varepsilon_{B_4}(n),\delta_{B_4})-r(n_{B_4},t_{B_4}(n_{B_4}),\varepsilon_{B_4}(n_{B_4}),\delta_{B_4}). \end{aligned} \tag{2.12}$$

According to our derivations in Scheme A4, it follows that

$$\begin{aligned} &r(n,t,\varepsilon,\delta)-r(n,t,\varepsilon,\delta_{B_4})\geqslant 0,\\ &r(n,t,\varepsilon,\delta_{B_4})-r(n,t_{B_4}(n),\varepsilon_{B_4}(n),\delta_{B_4})\geqslant 0,\\ &r(n,t_{B_4}(n),\varepsilon_{B_4}(n),\delta_{B_4})-r(n_{B_4},t_{B_4}(n_{B_4}),\varepsilon_{B_4}(n_{B_4}),\delta_{B_4})\geqslant 0. \end{aligned} \tag{2.13}$$

This proves the theorem. □

The following result guarantees the finiteness of the optimal sample size n_{B_i} for all cases of $i=1,2,3,4$.

Theorem 2.3. *Let n_{B_i} be the optimal sample size derived respectively through* Scheme A1 *previously defined*, $i = 1, 2, 3, 4$. *Then*,

$$n_{B_i} \leqslant \min\left(\frac{\varphi_g(0,0)}{C_1}, \frac{C_3}{C_1}\right) + \frac{C_2}{C_1} \quad \text{for } i = 1, 2, 3, 4$$

and

$$t_{B_i} \leqslant \min\left(\frac{\varphi_g(0,0)}{C_2}, \frac{C_3}{C_2}\right) + 2 \quad \text{for } i = 2, 4,$$

where $\varphi_g(0,0) = \int_0^\infty h(\lambda)g(\lambda)\,\mathrm{d}\lambda < \infty$ *by assumption.*

Proof. Let $(0, \delta_B(\,|0))$ denote the sampling plan for which no data is observed. According to (2.11),

$$\delta_B(\,|0) = \begin{cases} 1 & \text{if } \varphi_g(0,0) \leqslant C_3, \\ 0 & \text{otherwise.} \end{cases}$$

Consider the situation that both t and ε are prefixed. Then, according to (2.8) $r(0,\delta_B) = (t-\varepsilon)C_2 + C_3 + r_1(\delta_B|0)$. Since $\delta_B(\,|0) = 1_{(\varphi_g(0,0)\leqslant C_3)}$, hence, $C_3 + r_1(\delta_B|n) = \min(\varphi_g(0,0), C_3)$ according to (2.9) and (2.10). Thus, $r(0,\delta_B) = (t-\varepsilon)C_2 + \min(\varphi_g(0,0), C_3)$. Again, since δ_{B_1} is a Bayes solution, $r(n_{B_1}, \delta_{B_1}) \leqslant r(0, \delta_B)$, i.e. $(t - (\varepsilon/n_{B_1} + 1))C_2 + n_{B_1}C_1 + C_3 + r_1(\delta_{B_1}|n_{B_1}) \leqslant (t-\varepsilon)C_2 + \min(\varphi_g(0,0), C_3)$, we have $n_{B_1}C_1 + C_3 + r_1(\delta_{B_1}|n_{B_1}) \leqslant \min(\varphi_g(0,0), C_3)$ because $n_{B_1} \geqslant 0$ and $(\varepsilon/n_B + 1) \leqslant \varepsilon$. Now, since $C_3 \geqslant 0$ and $r_1(\delta_{B_1}|n_{B_1}) \geqslant 0$, hence, $n_{B_1} \leqslant \min(\varphi_g(0,0)/C_1, C_3/C_1)$.

If both t and ε are flexible, take any t and ε such that $t - \varepsilon = 1$. Then, $r(0,\delta_B) = C_2 + \min(\varphi_g(0,0), C_3)$. Again, since $r(n_{B_4}, \delta_{B_4}) \leqslant r(0, \delta_B)$, we have $(t_{B_4} - (\varepsilon_{B_4}/n_{B_4} + 1))\,C_2 + n_{B_4}\,C_1 + C_3 + r_1(\delta_{B_4}|n_{B_4}) \leqslant C_2 + \min(\varphi_g(0,0), C_3)$. So,

$$t_{B_4}C_2 \leqslant 2C_2 + \min(\varphi_g(0,0), C_3)$$

or

$$t_{B_4} \leqslant 2 + \min\left(\frac{\varphi_g(0,0)}{C_2}, \frac{C_3}{C_2}\right).$$

Also,

$$n_{B_4}C_1 + C_3 + r_1(\delta_{B_4}|n_{B_4}) \leqslant \min(\varphi_g(0,0), C_3) \quad \text{if } t_{B_4} - \frac{\varepsilon_{B_4}}{n_{B_4}+1} \geqslant 1,$$

and

$$n_{B_4}C_1 + C_3 + r_1(\delta_{B_4}|n_{B_4}) \leqslant \min(\varphi_g(0,0), C_3) + C_2 \quad \text{if } t_{B_4} - \frac{\varepsilon_{B_4}}{n_{B_4}+1} < 1.$$

In both cases, we can conclude that

$$n_{B_4} \leqslant \min\left(\frac{\varphi_g(0,0)}{C_1}, \frac{C_3}{C_1}\right) + \frac{C_2}{C_1}.$$

Same conclusion can be obtained for other situations by analogous argument. So the proof is complete.

It is to be noted that n_{B_i} and t_{B_i} are always finite, therefore in a finite steps through Schemes A1 previously defined, it is guaranteed that a Bayes solution $\delta_i(\cdot)$ can always be obtained for $i = 1, 2, 3, 4$.

3. Bayes sampling plan for quadratic loss

To obtain the Bayesian sampling plan (n_{B_i}, δ_{B_i}) for nonlinear loss, for simplicity, we assume $h(\lambda)$ to be a quadratic function $h(\lambda) = a_0 + a_1\lambda + a_2\lambda^2$ where a_0, a_1 and a_2 are all positive coefficients. Follow same assumption that prior distribution for scale parameter λ is a $\Gamma(\alpha, \beta)$ distribution.

A straightforward computation shows that for given $(\underset{\sim}{Z}(N), M) = (\underset{\sim}{z}(n), m)$, the posterior probability density of Λ is then

$$g(\lambda|z(n), m) \sim \Gamma(m + \alpha, z(n) + \beta).$$

We have

$$\varphi_g(z(n), m) = \int_0^\infty h(\lambda)\, g(\lambda|z(n), m)\, \mathrm{d}\lambda$$
$$= a_0 + \frac{a_1(m + \alpha)}{z(n) + \beta} + \frac{a_2(m + \alpha)(m + \alpha + 1)}{[z(n) + \beta]^2}, \tag{3.1}$$

and

$$\delta_{B_i}(\underset{\sim}{z}(n), m|n) = \begin{cases} 1 & \text{if } \varphi_g(z(n), m) \leqslant C_3, \\ 0 & \text{otherwise.} \end{cases} \tag{3.2}$$

Note that if $C_3 \leqslant a_0$, then $\varphi_g(z(n), m) > C_3$ for all $(\underset{\sim}{z}(n), m)$. Therefore $\delta_{B_i}(\underset{\sim}{z}(n), m|n) \equiv 0$. To avoid this extreme case, we assume that $C_3 > a_0$.

From (3.1) and (3.2) it follows that $\delta_{B_i}(\underset{\sim}{z}(n), m|n) = 1$ if, and only if,

$$(C_3 - a_0)[z(n) + \beta]^2 - a_1(m + \alpha)[z(n) + \beta] - a_2(m + \alpha)(m + \alpha + 1) \geqslant 0,$$

which is equivalent to

$$z(n) + \beta \geqslant \frac{a_1(m + \alpha) + \sqrt{a_1^2(m + \alpha)^2 + 4(C_3 - a_0)a_2(m + \alpha)(m + \alpha + 1)}}{2(C_3 - a_0)}$$
$$\equiv D_n(m)$$

say.

Thus, the Bayes decision function $\delta_{B_i}(\ |n)$ can be expressed as

$$\delta_{B_i}(\underset{\sim}{z}(n), m|n) = \begin{cases} 1 & \text{if } z(n) \geqslant D_n^*(m), \\ 0 & \text{otherwise,} \end{cases} \tag{3.3}$$

where $D_n^*(m) = D_n(m) - \beta$.

In the following it is desired to compute the Bayes risk associated with the Bayes decision given by (3.3). This Bayes risk will be derived and decomposed in four parts and each part is presented explicitly through (3.8)–(3.10).

Let $\mu_1 = E_g[\Lambda]$, $\mu_2 = E_g[\Lambda^2]$. Thus, $\int_0^\infty h(\lambda)g(\lambda)\,\mathrm{d}\lambda = a_0 + a_1\mu_1 + a_2\mu_2$.

Also, let

$$\Delta_m(n,\beta) = \Bigg\{ (z_1,\ldots,z_n) \mid 0 \leqslant z_i \leqslant z_j,\ i=1,\ldots,m;\ t-\varepsilon \leqslant z_j \leqslant t,$$

$$j = m+1,\ldots,n;\ \sum_{j=1}^{n} z_j < D_n^*(m) \Bigg\};$$

and

$$H(m,n,\beta) = \int \underset{\Delta_m(n,\beta)}{\cdots} \int \lambda^m \exp\left(-\lambda \sum_{i=1}^{n} z_i\right) \prod_{i=1}^{m} \left\{ \frac{t-z_i}{\varepsilon} \wedge 1 \right\} \mathrm{d}z_1 \cdots \mathrm{d}z_m.$$

The Bayes risk for the sampling plan $(n, \delta_{\mathrm{B}_i}(|n))$ due to (2.8) can be computed as following and it finally can be decomposed into four exclusive parts.

$$\begin{aligned} r(n,\delta_{\mathrm{B}_i}) &= nC_1 + \left(t - \frac{\varepsilon}{n+1}\right) C_2 + \int_0^\infty h(\lambda)g(\lambda)\,\mathrm{d}\lambda \\ &\quad + E\{[C_3 - h(\Lambda)][1 - \delta_{\mathrm{B}_i}(\underset{\sim}{Z}(N), M|n)]\} \\ &= nC_1 + \left(t - \frac{\varepsilon}{n+1}\right) C_2 + a_0 + a_1\mu_1 + a_2\mu_2 \\ &\quad + \int_0^\infty [C_3 - h(\lambda)] P\{\delta_{\mathrm{B}_i}(\underset{\sim}{Z}(N), M|n) = 0|\lambda\} g(\lambda)\,\mathrm{d}\lambda, \end{aligned} \tag{3.4}$$

where

$$\begin{aligned} P\{\delta_{\mathrm{B}_i}(\underset{\sim}{Z}(N), M|n) = 0|\lambda\} &= P\left\{ \sum_{j=1}^{N} Z_j < D_n(M) - \beta | \lambda \right\} \\ &= P\{M = 0|\lambda\}\, I(nt < D_n(0) - \beta) \\ &\quad + \sum_{m=1}^{n} \int \underset{\Delta_m(n,\beta)}{\cdots} \int \binom{n}{m} \lambda^m \exp\left(-\lambda \sum_{i=1}^{n} z_i\right) \\ &\quad \prod_{i=1}^{m} \left\{ \frac{t-z_i}{\varepsilon} \wedge 1 \right\} \mathrm{d}z_1 \cdots \mathrm{d}z_m \end{aligned}$$

$$= P\{M = 0|\lambda\}\, I(nt < D_n(0) - \beta) + \sum_{m=1}^{n} \binom{n}{m} H(m,n,\beta). \tag{3.5}$$

In the following each component of the Bayes risk will be computed explicitly.

Let $[x]$ denote the largest integer not exceeding x. From Theorem A.3 in the appendix of Lam and Choy (1995), we can express

$$H(m,n,\beta) = \begin{cases} 0 & \text{if } D_n^*(m) < (n-m)(t-\varepsilon), \\ \lambda^m \displaystyle\sum_{j=0}^{D_{D_n^*(m)}} \sum_{k=0}^{E_{j,D_n^*(m)}} \binom{m}{j} \binom{n-m+j}{k} & \text{if } (n-m)(t-\varepsilon) \leqslant D_n^*(m) \\ \dfrac{(-1)^{j+k}}{\varepsilon^j (n+j-1)!} & \quad = D_n^*(m) \leqslant nt, \\ \times \displaystyle\int_0^{D_n^*(m)-d} u^{n+j-1} \exp\{-\lambda(u+d)\}\,\mathrm{d}u, & \\ \displaystyle\sum_{j=0}^{m} \binom{m}{j} (-1)^j \varepsilon^{n-m} \exp\{-(n-m+j)\lambda t\} & \\ \times \left(\dfrac{\exp(\lambda\varepsilon)-1}{\lambda\varepsilon}\right)^{n-m+j} & \text{if } D_n^*(m) > nt, \end{cases} \tag{3.6}$$

where $d = (n-m+j)(t-\varepsilon) + k\varepsilon$, $D_{D_n^*(m)} = \min\{[D_n^*(m)/(t-\varepsilon)] - n + m, m\}$, and $E_{j,D_n^*(m)} = \min\{[D_n^*(m) - (n-m+j)(t-\varepsilon)/\varepsilon], n-m+j\}$.

Let $I_n = \{1,\ldots,n\}$ and let

$$A \equiv A(n,t,\beta) = \{m \in I_n \mid D_n^*(m) < (n-m)(t-\varepsilon)\}$$

$$B \equiv B(n,t,\beta) = \{m \in I_n \mid (n-m)(t-\varepsilon) < D_n^*(m) \leqslant nt\},$$

$$C \equiv C(n,t,\beta) = \{m \in I_n \mid D_n^*(m) > nt\}.$$

By the definition of $D_n(m)$, we see that both $D_n(m) - \beta - nt$ and $D_n(m) - \beta - (n-m)(t-\varepsilon)$ are increasing in m for $m \in I_n$. Suppose that A, B and C are nonempty. Then, for any m_1 in A, m_2 in B and m_3 in C, we must have $m_1 < m_2 < m_3$. According to (3.6), for $m \in A$, $H(m,n,\beta) = 0$. For $m \in C$,

$$H(m,n,\beta) = \sum_{j=0}^{m} \binom{m}{j} (-1)^j \varepsilon^{n-m} \exp\{-(n-m+j)\lambda t\} \left(\frac{\exp(\lambda\varepsilon)-1}{\lambda\varepsilon}\right)^{n-m+j},$$

and for $m \in B$,

$$H(m,n,\beta) = \lambda^m \sum_{j=0}^{D_{D_n^*(m)}} \sum_{k=0}^{E_{j,D_n^*(m)}} \binom{m}{j} \binom{n-m+j}{k} \frac{(-1)^{j+k}}{\varepsilon^j (n+j-1)!}$$

$$\times \int_0^{D_n^*(m)-d} u^{n+j-1} \exp\{-\lambda(u+d)\}\,\mathrm{d}u.$$

Therefore, the Bayes risk $r(n,\delta_{\mathrm{B}_i}(\,|n))$ can be expressed as

$$\begin{aligned} r(n,\delta_{\mathrm{B}_i}(\,|n)) = {} & \left[nC_1 + \left(t - \frac{\varepsilon}{n+1}\right) C_2 + a_0 + a_1\mu_1 + a_2\mu_2\right] \\ & + \int_0^\infty [C_3 - h(\lambda)] P\{M=0|\lambda\} I(nt < D_n(0) - \beta) g(\lambda)\,\mathrm{d}\lambda \\ & + \sum_{m\in B} \int_0^\infty [C_3 - h(\lambda)] \binom{n}{m} H(m,n,\beta) g(\lambda)\,\mathrm{d}\lambda \\ & + \sum_{m\in C} \int_0^\infty [C_3 - h(\lambda)] \binom{n}{m} H(m,n,\beta) g(\lambda)\,\mathrm{d}\lambda \\ \equiv {} & r_1 + r_2 + r_3 + r_4, \end{aligned} \tag{3.7}$$

where $r_1 = nC_1 + (t - \varepsilon/(n+1))C_2 + a_0 + a_1\mu_1 + a_2\mu_2$.

Note that $P\{M=0|\lambda\} = \exp\{-\lambda nt\}$. A straightforward computation shows that

$$\begin{aligned} r_2 &= I(nt < D_n(0) - \beta) \int_0^\infty [C_3 - a_0 - a_1\lambda - a_2\lambda^2] \mathrm{e}^{-\lambda nt} \frac{\beta^\alpha \lambda^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda\beta}\,\mathrm{d}\lambda \\ &= I(nt < D_n(0) - \beta) \left\{ \frac{(C_3 - a_0)\beta^\alpha}{(nt+\beta)^\alpha} - \frac{a_1\alpha\beta^\alpha}{(nt+\beta)^{\alpha+1}} - \frac{a_2\alpha(\alpha+1)\beta^\alpha}{(nt+\beta)^{\alpha+2}} \right\}. \end{aligned} \tag{3.8}$$

Following a discussion analogous to (2.12)–(2.13) of Lam and Choy (1995), we can obtain

$$\begin{aligned} r_3 &= E\Bigg\{ (C_3 - a_0 - a_1\lambda - a_2\lambda^2) \\ &\quad \times \sum_{m\in B} \int \cdots \int f(z_1,1,\ldots,z_m,1,z_{m+1},0,\ldots,z_n,0;m)\,\mathrm{d}z_1\cdots\mathrm{d}z_n \Bigg\} \\ &= E\left\{ (C_3 - a_0 - a_1\lambda - a_2\lambda^2) \sum_{m\in B} \binom{n}{m} \frac{1}{\varepsilon^{n-m}} H(m,n,\beta) \right\} \end{aligned}$$

$$
= E\left\{(C_3 - a_0 - a_1\lambda - a_2\lambda^2)\lambda^m \sum_{m\in B}\sum_{j=0}^{D_{D_n^*(m)}}\sum_{k=0}^{E_{j,D_n^*(m)}}\binom{n}{m}\binom{m}{j}\binom{n-m+j}{k}\right.
$$

$$
\left.\times\frac{(-1)^{j+k}}{\varepsilon^{n-m+j}(n+j-1)!}\int_0^{D_n^*(m)-d} u^{n+j-1}\exp\{-\lambda(u+d)\}\,\mathrm{d}u\right\}
$$

$$
= \sum_{m\in B}\sum_{j=0}^{D_{D_n^*(m)}}\sum_{k=0}^{E_{j,D_n^*(m)}}\binom{n}{m}\binom{m}{j}\binom{n-m+j}{k}\frac{(-1)^{j+k}\beta^\alpha}{\varepsilon^{n-m+j}(n+j-1)!\Gamma(\alpha)}
$$

$$
\times\{(C_3-a_0)\Gamma(m+\alpha)\xi_{m+\alpha} - a_1\Gamma(m+\alpha+1)\xi_{m+\alpha+1}
$$

$$
- a_2k\Gamma(m+\alpha+2)\xi_{m+\alpha+2}\}, \tag{3.9}
$$

where d, $D_{D_n^*(m)}$ and $E_{j,D_n^*(m)}$ are respectively defined in (3.6) and

$$
\xi_r = \int_0^{D_n^*(m)-d}\frac{u^{n+j-1}}{(u+d+\beta)^r}\,\mathrm{d}u = \sum_{i=0}^{n+j-1}\binom{n+j-1}{i}(-1)^i(d+\beta)^i
$$

$$
\times\int_0^{D_n^*(m)-d}(u+d+\beta)^{n+j-i-r-1}\,\mathrm{d}u
$$

for $r = m+\alpha, m+\alpha+1, m+\alpha+2$.

Obviously, ξ_r can be integrated analytically. Moreover, analogous to (2.14) of Lam and Choy (1995), we have

$$
r_4 = \sum_{m\in C}\sum_{j=0}^{m}\binom{n}{m}\binom{m}{j}\frac{(-1)^j\beta^\alpha}{\Gamma(\alpha)}\int_0^\infty (C_r - a_0 - a_1\lambda - a_2\lambda^2)\lambda^{\alpha-1}
$$

$$
\times\exp\{-(n-m+j)\lambda t - \beta\lambda\}\left(\frac{\exp(\lambda\varepsilon)-1}{\lambda\varepsilon}\right)^{n-m+j}\mathrm{d}\lambda. \tag{3.10}
$$

Here the value of r_4 cannot be straightforwardly evaluated analytically in general and a numerical method can be used for the computation of its value.

Combining (3.7)–(3.10), an explicit presentation of the Bayes risk of the sampling plan (n,δ) is thus derived.

It is obvious through (3.3) that, under FCT scheme and taking $C_2 = 0$ in loss, if optimal solution (n_0, T_0) of Lam and Choy (1995) is so chosen that $n_0 = n_{\mathrm{B}_1}$ and $T_0 = D_n^*(m)$, then their optimal solution is a Bayes solution. However, it is readily seen that in general their optimal solution for T_0 is not $D_n^*(m)$ given by (3.3), so it is not Bayes.

4. Algorithm for optimal solution

Based on the Bayes risk, a simple algorithm using following steps can be used to obtain an optimal sampling plan. In the following we denote n^* and t^*, respectively, to be the upper bound of n and t for each censoring scheme. I_n is defined by (3.6).

Algprithm B.

(1) Start with $n = 0$, compute $r(0, 0)$.

(2a) Censoring scheme is FCT.
For each $n = 1, \ldots, n^*$, compute $r(n, \delta)$ and minimize $r(n, \delta)$ with respect to δ. We denote the minimizer by δ_{B_1}.

(2b) Censoring scheme is t-FCT.
For each $n = 1, \ldots, n^*$, compute $r(n, \delta)$ and minimize $r(n, \delta)$ with respect to δ and t. We denote, respectively, the minimizer by δ_{B_2} and t_{B_2}.

(2c) Censoring scheme is ε-FCT.
For each $n = 1, \ldots, n^*$, compute $r(n, \delta)$ and minimize $r(n, \delta)$ with respect to δ and ε. We denote, respectively, the minimizer by δ_{B_3} and ε_{B_3}.

(2d) Censoring scheme is FUCT.
For each $n = 1, \ldots, n^*$, compute $r(n, \delta)$ and minimize $r(n, \delta)$ with respect to δ, t and ε. We denote, respectively, the minimizer by δ_{B_4}, t_{B_4} and ε_{B_4}.

(3) Compare the risks among $r(0, 0)$ and $r(n, \delta_{B_i})$. Let $S = \{n \in I_{n^*} \mid r(n, \delta_{B_i}) < r(0, 0)\}$.
Then, for $i = 1, 2, 3, 4$, n_{B_i}, is determined as

$$n_{B_i} = \begin{cases} 0 & \text{if } S = \phi, \\ \min\{n \mid n \in S\} & \text{if } S \neq \phi. \end{cases} \tag{4.1}$$

Numerical approximation C

First let $L(N, t^*) = t^*/N$ where $t^* = 2$. Take $t_j \equiv t_j(N, t^*) = (j - 0.5)L(N, t^*)$, $\varepsilon_j = 0.0001(0.0002)t_j$, $j = 1, \ldots, N$, for $0 < \varepsilon \leqslant t \leqslant t^*$, $N = 60\,000$. Let I_N be defined in (3.6).

(1) *t-FCT scheme*
For each n, compute $r(n, \delta_{B_2})$ and take

$$t_{B_2}(n) = \min\left\{t_i \mid i \in I_N,\ r(n, \delta_{B_2}) = \min_{1 \leqslant j \leqslant N}\{r(n, \delta_{B_2})\ \forall t_i \geqslant \varepsilon > 0\}\right\}.$$

(2) *ε-FCT scheme*
For each n, compute $r(n, \delta_{B_3})$ and take

$$\varepsilon_{B_3}(n) = \min\left\{\varepsilon_i \mid i \in I_N, r(n, \delta_{B_3}) \min_{1 \leqslant j \leqslant N}\{r(n, \delta_{B_3})\ \forall t \geqslant \varepsilon_j > 0\}\right\}.$$

(3) *FUCT scheme*

For each n, compute $r(n, \delta_{B_4})$ and take the pair

$$(t_{B_4}, \varepsilon_{B_4}) = \min \left\{ (t_i, \varepsilon_j) \mid i, j \in I_N, r(n, \delta_{B_4}) \right.$$

$$\left. = \min_{1 \leqslant j \leqslant N,\ 1 \leqslant i \leqslant N} \{ r(n, \delta_{B_4})\ \forall t_i \geqslant \varepsilon_j > 0 \} \right\}.$$

To illustrate the proposed Bayes plan using the Algorithm B proposed in this section, some numerical examples are studied under quadratic loss. For its convenience for comparisons, here we take same constants as that in Lam and Choy (1995), so we take $\alpha = 3.0$, $\beta = 2.0$, $t = 2$, $\varepsilon = 1$, $a_0 = 20.0$, $a_1 = 5.0$, $a_2 = 10.0$, $C_1 = 0.5$, $C_3 = 50$ and $C_2 = 0$ (see Table 5). For other cases, we take $C_2 = 0.5$. In each table one coefficient is permitted to vary and the others are kept fixed. Here (n_{B_i}, δ_{B_i}) denotes optimal sampling plan, while $r(n_{B_i}, \delta_{B_i})$ is its Bayes risk under various situations as defined in Algorithm B.

For instance, under FCT scheme (Table 1), corresponding to $(\alpha, \beta, t, \varepsilon, a_0, a_1, a_2, C_1, C_2, C_3) = (2.5, 2, 2, 1, 20, 5, 10, 0.5, 0.5, 50)$ the optimal sampling plan (n_{B_1}, δ_{B_1}) is given by $(n_{B_1}, D_n^*(m)) = (2, 1.2717)$ which means 2 items are taken from the batch for inspection and accept the batch if the total length of observed lifed times $(z(n) \equiv \sum_{i=1}^n z_i)$ is no less than $D_n^*(m) = 1.2717$ (see (3.3)). Its Bayes risk is 42.0310. Also, for the FUCT scheme (Table 4), corresponding to $(\alpha, \beta, a_0, a_1, a_2, C_1, C_2, C_3) = (3.5, 2, 20, 5, 10, 0.5, 0.5, 50)$ the optimal sampling plan $(n_{B_4}, t_{B_4}, \varepsilon_{B_4}, \delta_{B_4})$ is given by $(n_{B_4}, t_{B_4}, \varepsilon_{B_4}, D_n^*(m)) = (2, 1.9758, 0.4518, 1.2717)$ which means that 2 items are drawn from the batch for inspection and the censoring time follows a uniform distribution U(1.9758–0.4518, 1.9758). The batch is accepted if the total length of observed life times is no less than 1.2717. It Bayes risk is 43.5541. For some optimal solution (n_{B_i}, δ_{B_i}), if $n_{B_i} = 0$, the total length of observed life times is 0 and thus the batch is rejected if its associated $D_n^*(m) > 0$, otherwise the batch is accepted.

It is easy to see that $r(n_{B_4}, \delta_{B_4}) \leqslant r(n_{B_1}, \delta_{B_1})$, so the scheme FUCT is always more favorable than FCT to the experimenter in the sense of its Bayes risk. However, for the comparison between scheme t-FCT and scheme ε-FCT, it depends on values of those parameters $\alpha, \beta, a_0, \ldots$, etc. As can be seen from entries of Tables 2 and 3, sometimes t-FCT is more favorable to an experimenter, sometimes ε-FCT is more favorable in the sense of its Bayes risk. However, it is to be noted that a censoring scheme is to be chosen beforehand by an experimenter which is supposed to be most appropriate to him.

In Table 5, we tabulate both the optimal solutions and its risks from Lam and Choy (1995) and the proposed Bayes solution of (3.3) in Section 3 taking exactly the same constants of Lam and Choy (1995), i.e. $(\alpha, \beta, t, \varepsilon, a_0, a_1, a_2, C_1, C_2, C_3) = (3.0, 2.0, 2, 1, 20, 5.0, 10, 0.5, 0, 50)$. Again, it shows that the optimal solution of Lam and Choy (1995) is not a Bayes solution.

In Figs. 1 and 2, Bayes risks $r(n, \delta_{B_1})$ are plotted with respect to n for various values of C_1 which keeping $\varepsilon = 0.25$ and 2.0, respectively, and other parameters

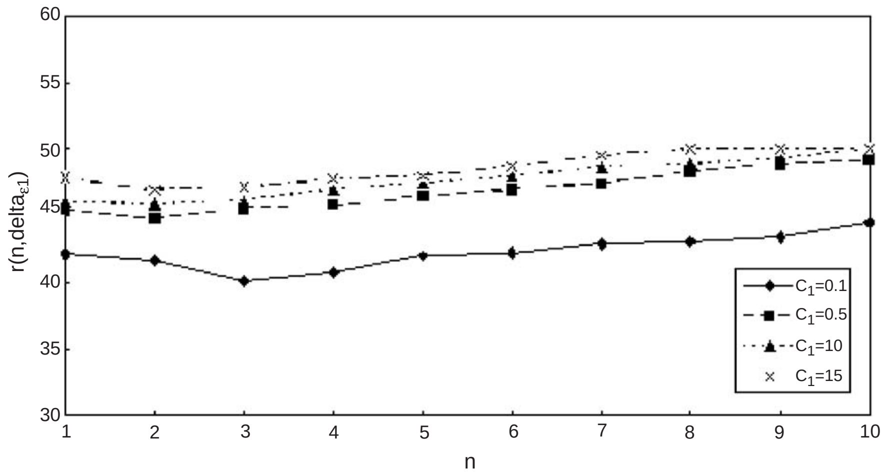

Fig. 1. Under FCT scheme, $\varepsilon = 0.25$, plots of Bayes risk r for various C_1 keeping other parameters fixed.

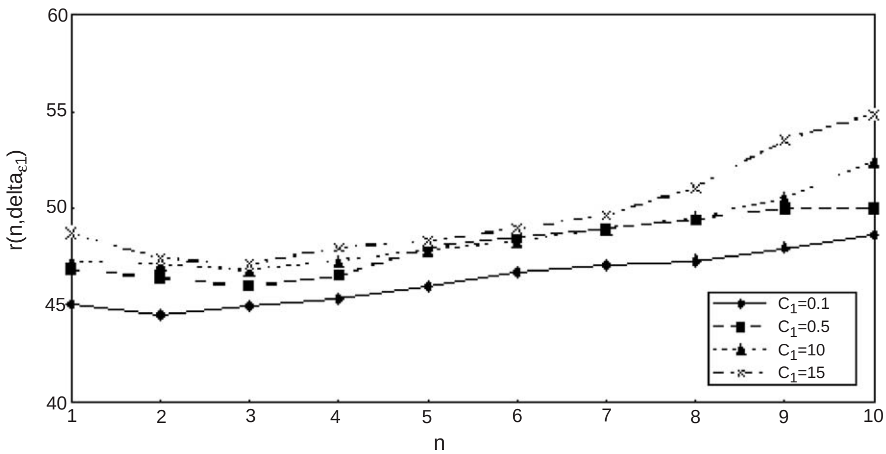

Fig. 2. Under FCT scheme, $\varepsilon = 2.0$, plots of Bayes risk r for various C_1 keeping other parameters fixed.

$(\alpha, \beta, t, a_0, a_1, a_2, C_2, C_3) = (3.0, 2.0, 2, 20, 5, 10, 0.5, 60)$. In Figs. 3 and 4, Bayes risks $r(n, \delta_{\mathrm{B}_2})$ are plotted with respect to n and C_1 respectively under t-FCT scheme keeping $\varepsilon = 0.25$, $n = 10$ (for Fig. 4), $C_1 = 0.5$, $C_2 = 60$ (Fig. 3) and other parameters fixed as in Fig. 1.

5. Conclusion

Lam and Choy's (1995) model is reconsidered under a general Bayes set up for more general situation of censoring. Four types of data are respectively considered

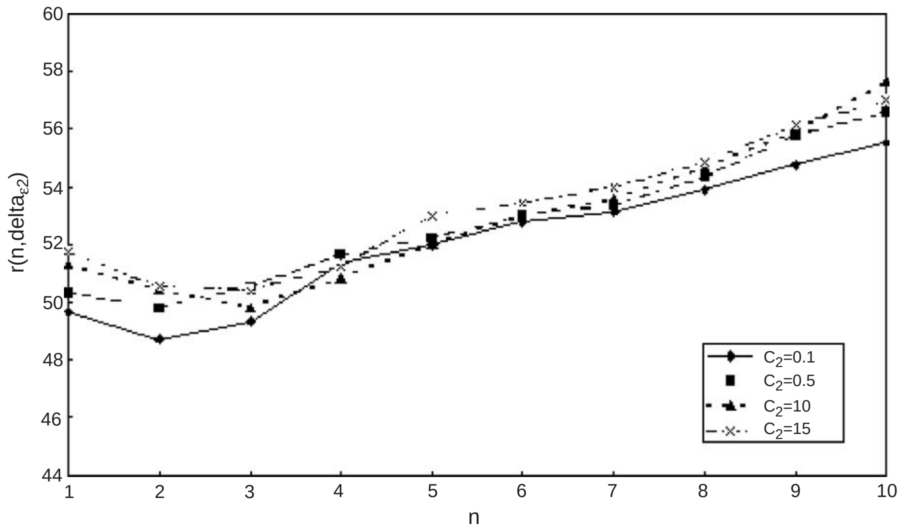

Fig. 3. Under t-FCT scheme, $\varepsilon = 0.25$, $C_1 = 0.5$, $C_3 = 60$, plots of Bayes risk r for various C_2 keeping other parameters fixed.

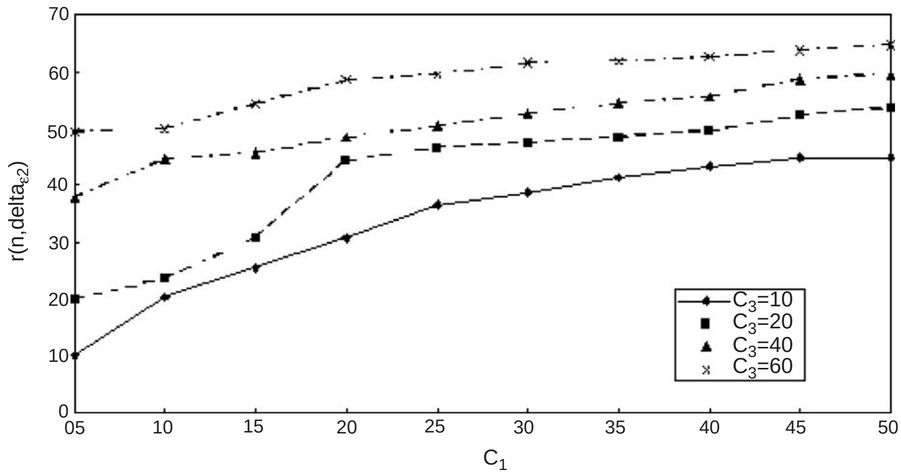

Fig. 4. Under t-FCT scheme, $\varepsilon = 0.25$, $C_2 = 0.5$, $n = 10$, plots of Bayes risk r with respect to C_1 for various C_3 keeping other parameters fixed.

for uniform random censoring. Censoring time t and length of the support of uniform distribution ε are respectively considered as either fixed or as a parameter. Cost of unit time for experiment is also included in the loss function. A Bayes sampling plan has been proposed under general setting for various situations of censoring and an explicit Bayesian sampling has been derived for a quadratic loss. Scheme A and Algorithm B are proposed to find the optimal Bayes sampling plan. Some optimal Bayes plans and its Bayes risks are tabulated (Tables 1–5). Some Bayes risks are also plotted for some special parameters. It has been shown that for the special situation (both t and ε are fixed) the sampling plan proposed by Lam and Choy (1995) is not Bayes.

It should be pointed out that the Bayes risk is not a smooth function of those variables involved, therefore numerical computations for finding optimal solutions are quite sensitive to computing method. To strengthen its accuracy of the numerical approximation, we take $N = 60\,000$ for division of $[0, t^*]$, which is much bigger than that of the Lam and Choy's case.

To extend the life model under consideration, it is natural to consider Weibull, IFR or more general model. The Bayes rule proposed in (2.11) can be analogously applied for general model, however, its computaion may be laborious.

Acknowledgements

We are grateful to an associate Editor and two referees for their helpful comments which have improved present presentation.

References

Ebrahimi, N., Habibullah, M., 1992. Testing to determine the underlying distribution using incomplete observations when the life time distribution is proportionally related to the censoring time distribution. J. Statist. Comput. Simulation 40, 109–118.

Fertig, K.W., Mann, N.R., 1974. A decision-theoretic approach to defining variables sampling plans for finite lots: single sampling for exponential and Gaussian process. J. Amer. Statist. Assoc. 69, 665–671.

Hald, A., 1967. Asymptotic properties of Bayesian single sampling plans. J. Roy. Statist. Soc. Ser. B 29, 162–173.

Hald, A., 1981. Statistical Theory of Sampling Inspection by Attributes. Academic, New York.

Köllerström, J., Wetherill, G.B., 1981. Sampling by variables with a normal-gamma prior. In: Lenz, H.-J., Wilrich, P.-T. (Eds.), Frontiers in Statistical Quality Control. Physica-Verlag, Würzburg, Germany.

Lam, Y., 1988a. A decision theory approach to variable sampling plans. Scienta Sinica Ser. A 31, 129–140.

Lam, Y., 1988b. Bayesian approach to single variable sampling plans. Biometrika 75, 387–391.

Lam, Y., 1994. Bayesian variable sampling plans for the exponential distribution with type I censoring. Ann. Statist. 22 (2), 696–711.

Lam, Y., Choy, S.T.B., 1995. Bayesian variable sampling plans for the exponential distribution with uniformly distributed random censoring. J. Statist. Plann. Inference 47, 277–293.

Lam, Y., Lau, L.C., 1993. Optimal single variable sampling plans. Commun. Statist. Simulation Comput. 22, 371–386.

Lehmann, E.L., 1959. Testing Statistical Hypotheses. Wiley, New York.

Wetherill, G.B., 1977. Sampling Inspection and Quality Control, 2nd Edition. Chapman & Hall, London.

Wetherill, G.B., Campling, G.B.G., 1966. The decision theory approach to sampling inspection. J. Roy. Statist. Soc. Ser. B 28, 381–416.

Wetherill, G.B., Köllerström, J., 1979. Sampling inspection simplified. J. Roy. Statist. Soc. Ser. A 142, 1–32 (with discussion).